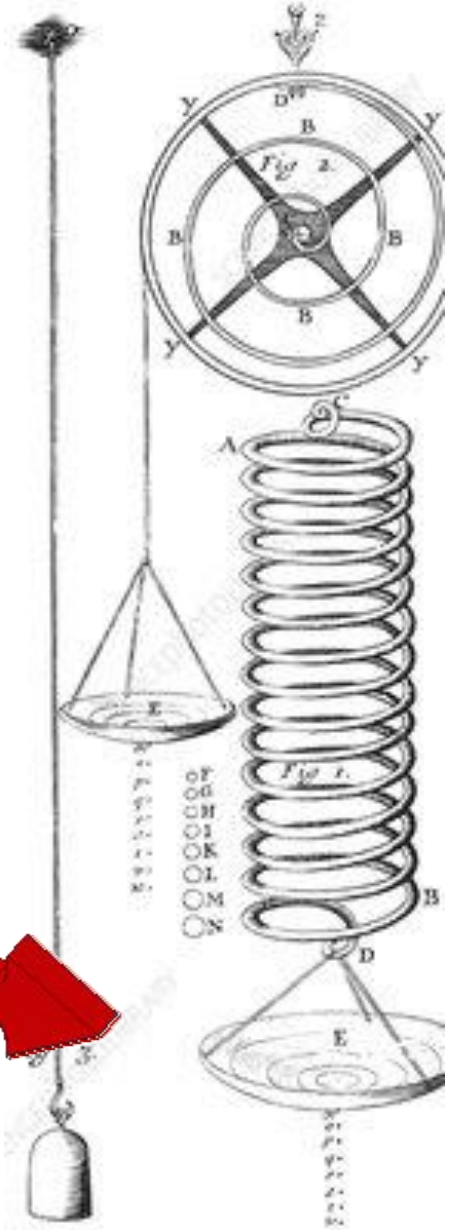
# Hooke’s Law

Ut tensio, sic vis,

- “The more you pull a spring the further it stretches”

- Force is proportional to change in length
- F = K(x)  where F = Force, K is stiffness and x is change in length



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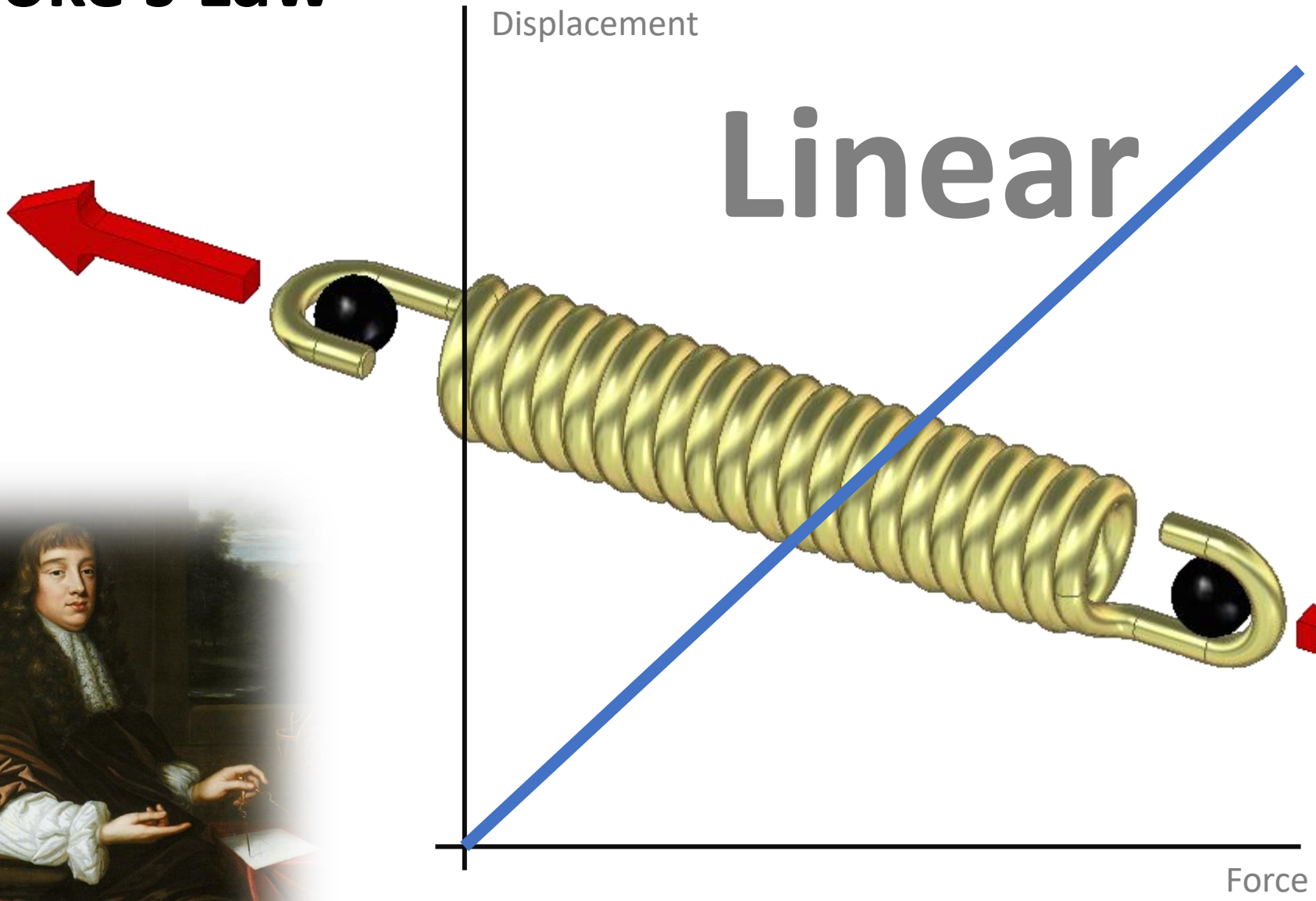
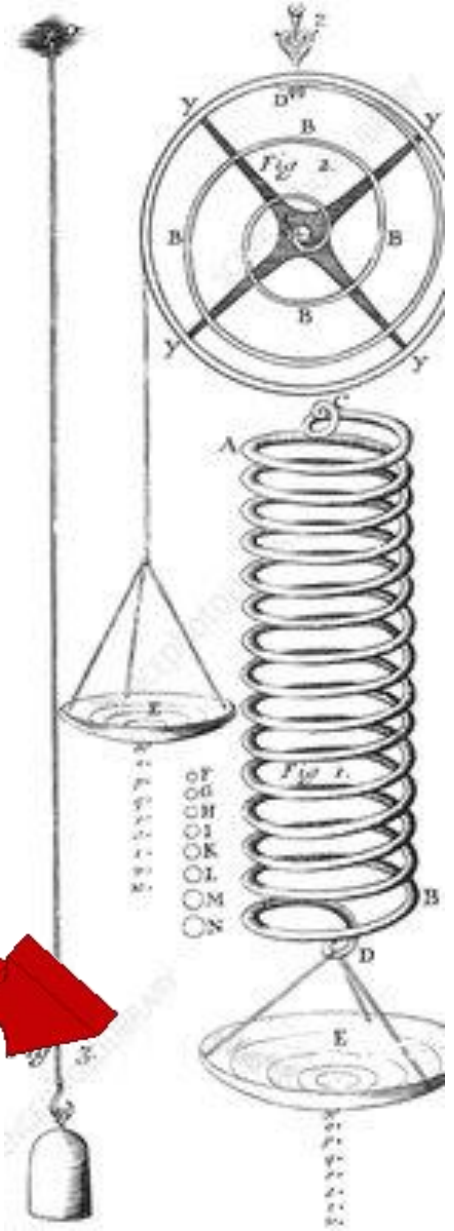
# Hooke’s Law





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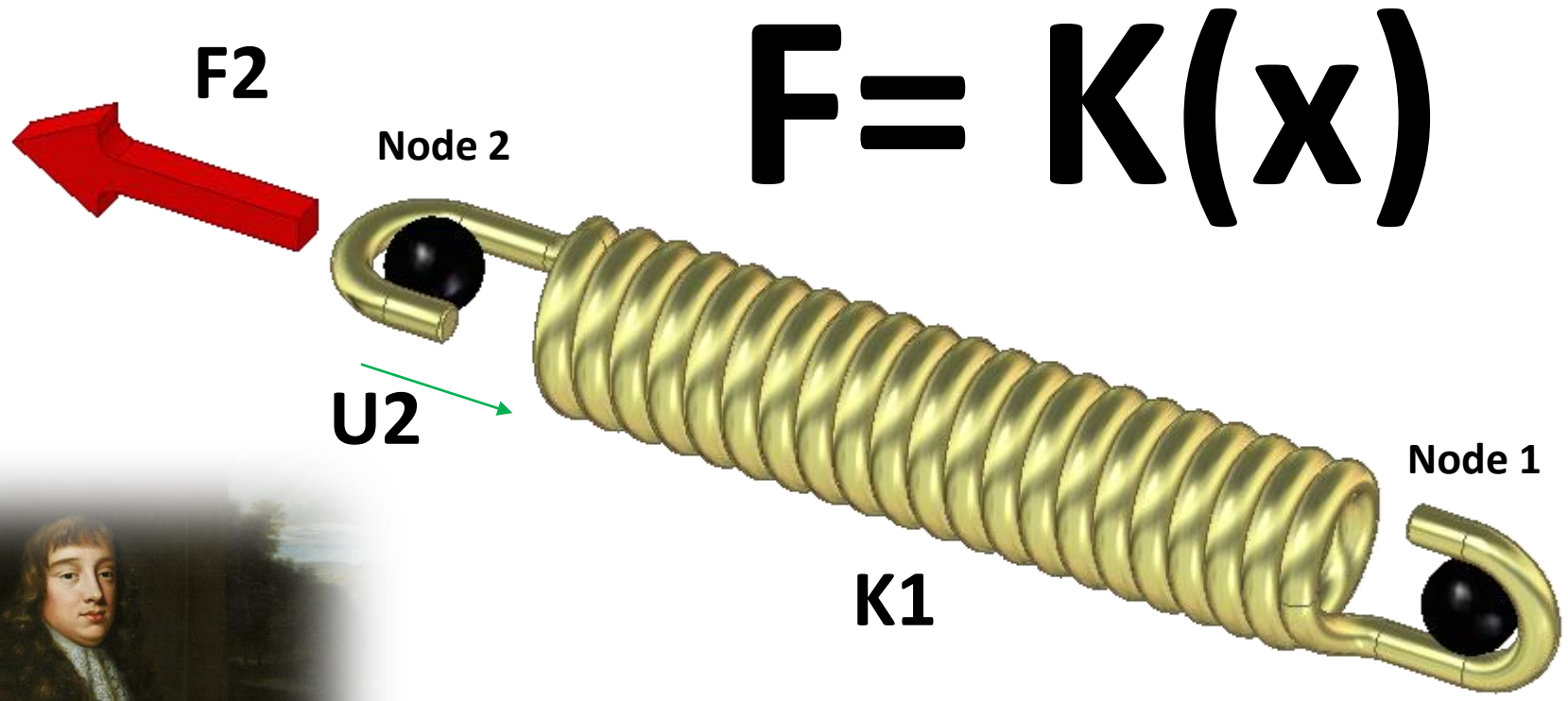
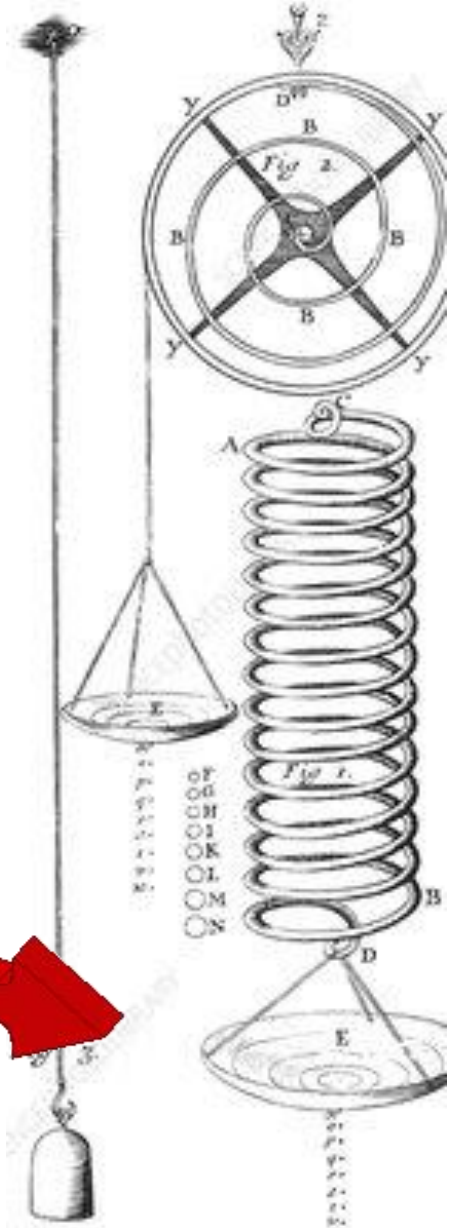
# Hooke's Law in Matrix form

$$F = K(x)$$




$$\begin{bmatrix} F1 \\ F2 \end{bmatrix} = \begin{bmatrix} K1 & -K1 \\ -K1 & K1 \end{bmatrix} \begin{bmatrix} U1 \\ U2 \end{bmatrix}$$

U1/U2 – nodal displacements
F1/F2 – nodal forces
K1 – element stiffness



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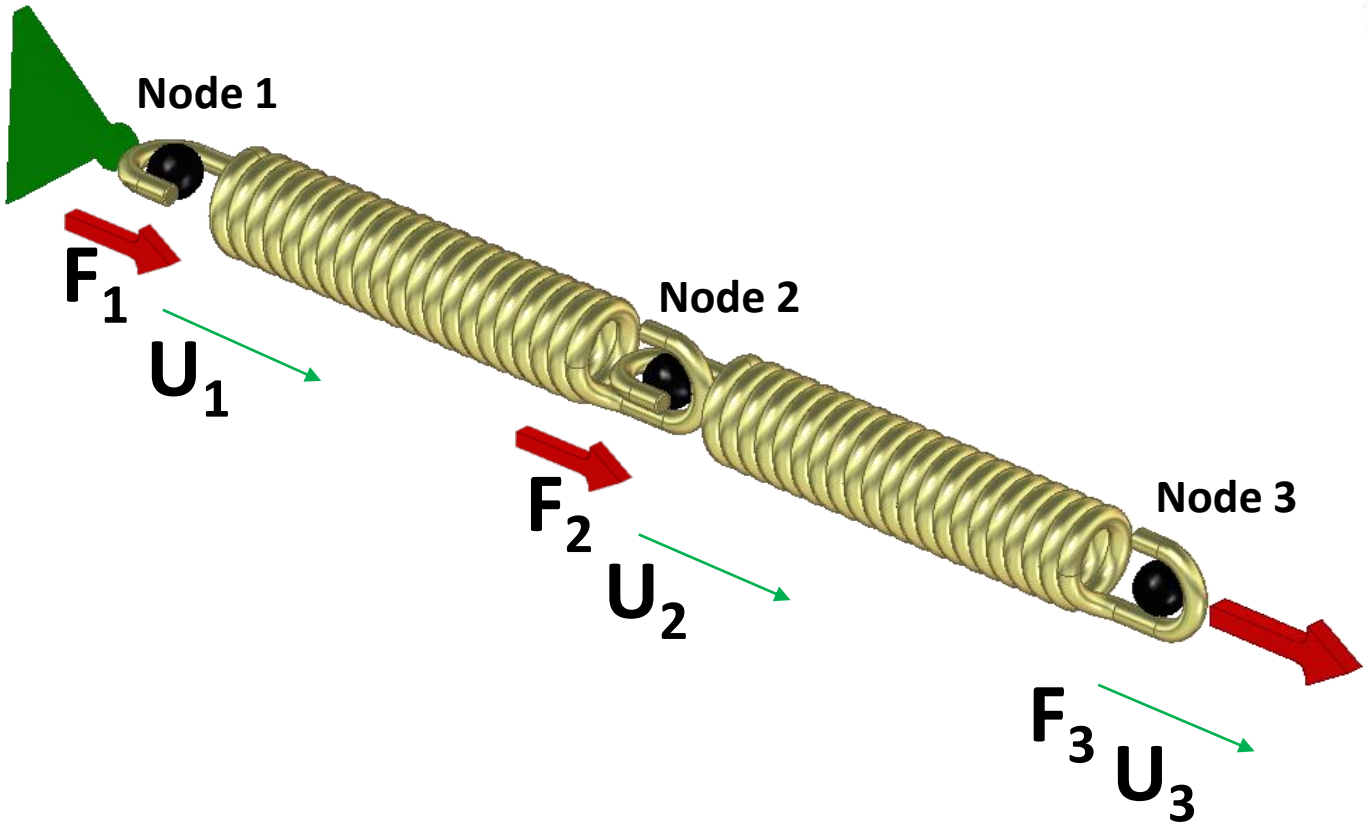
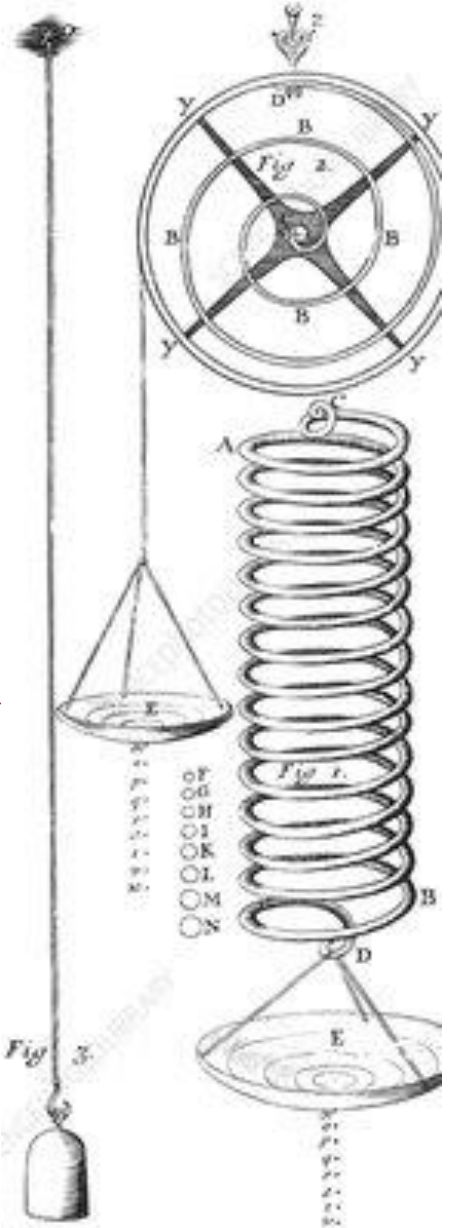
# Two springs




$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} K_1 & -K1 & 0 \\ -K_1 & K_1+K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$



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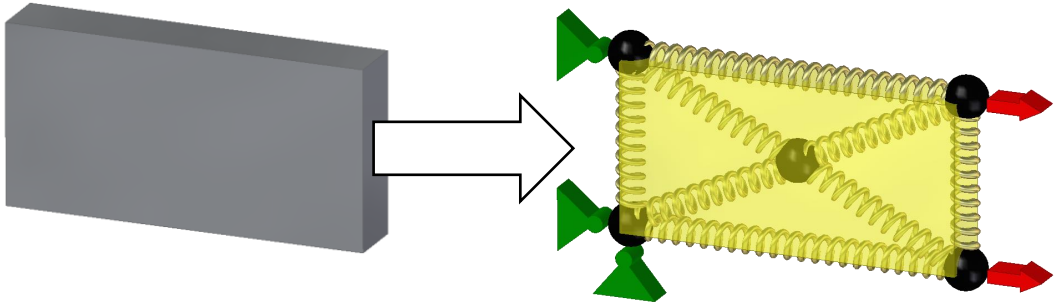
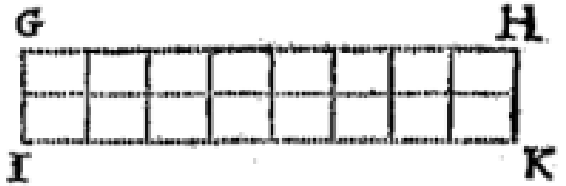
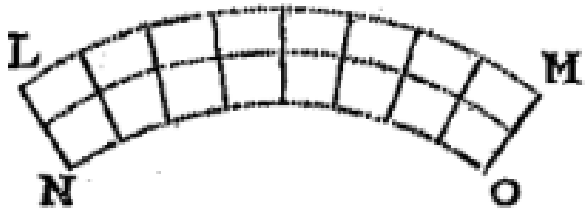
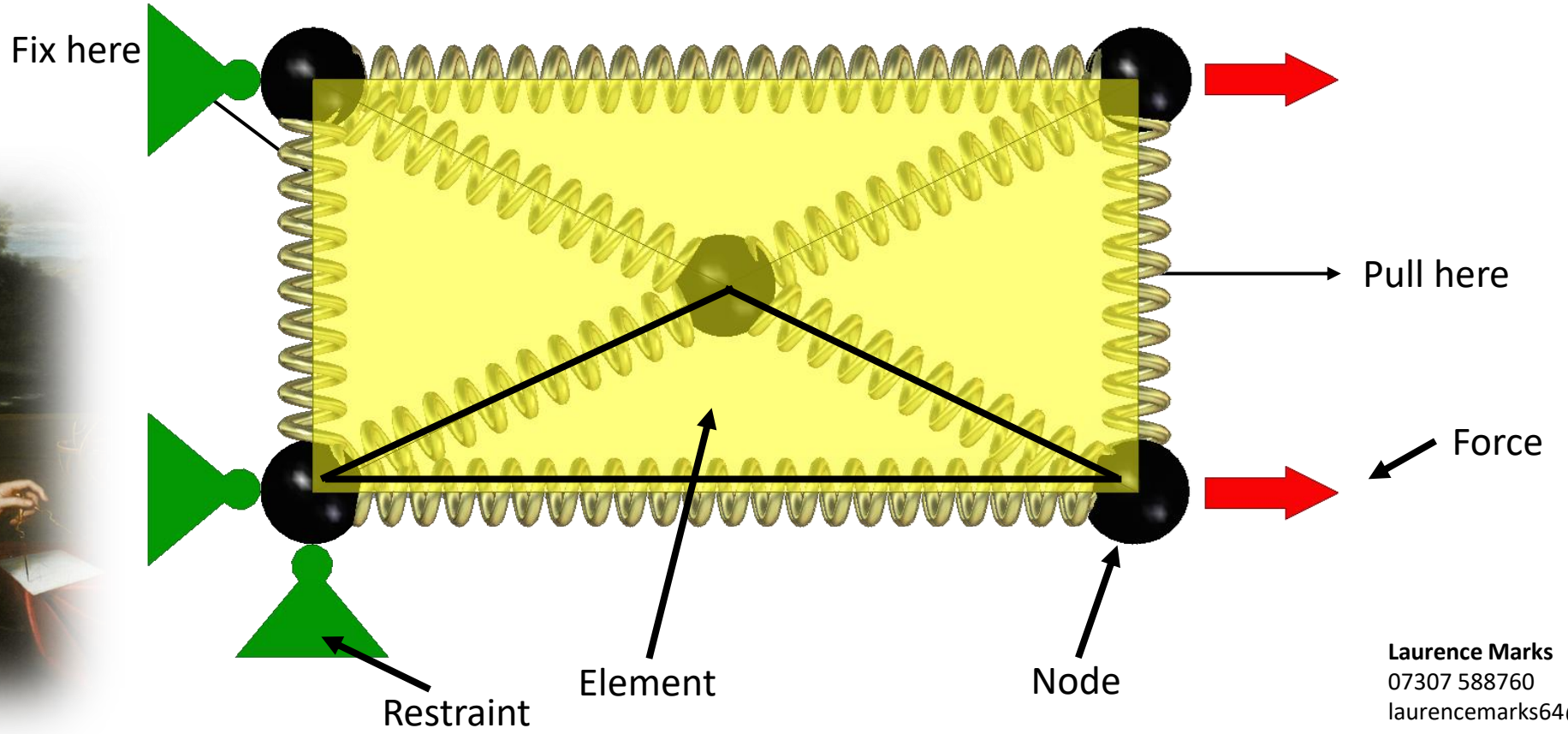
# Modelling Continuous Structures



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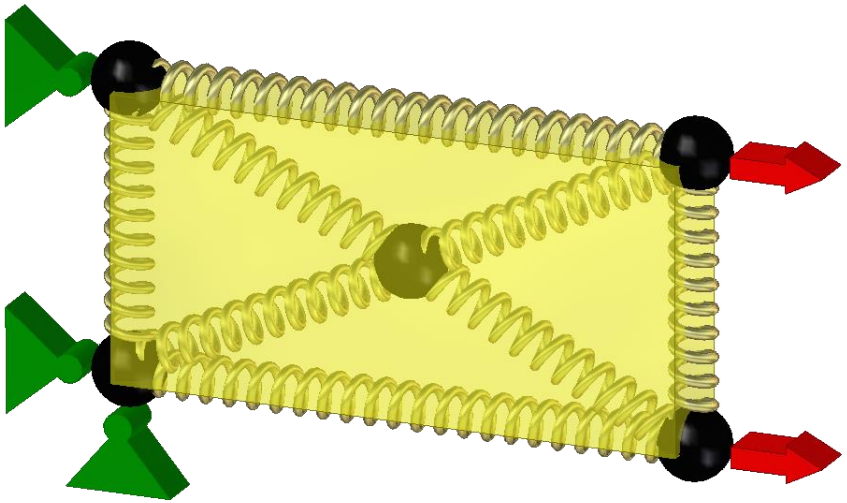
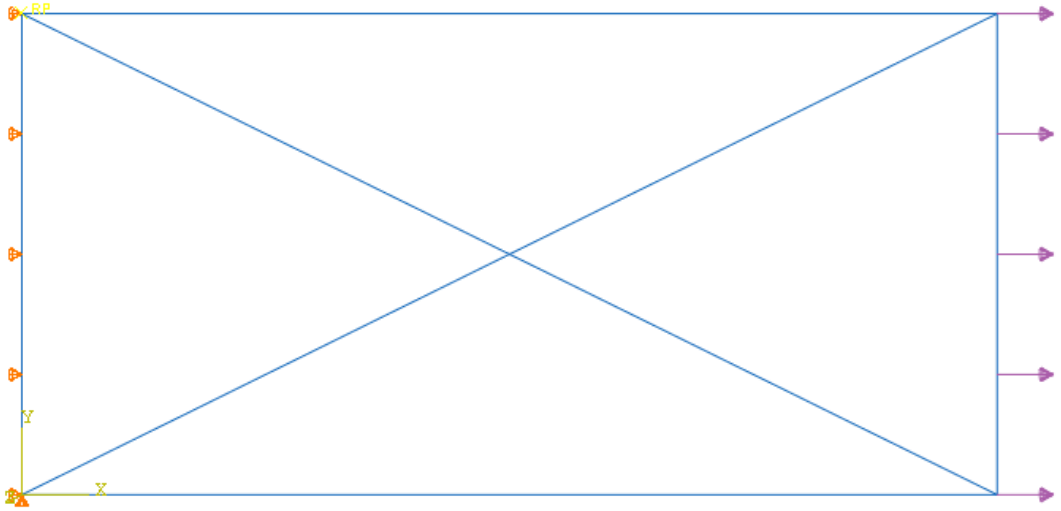
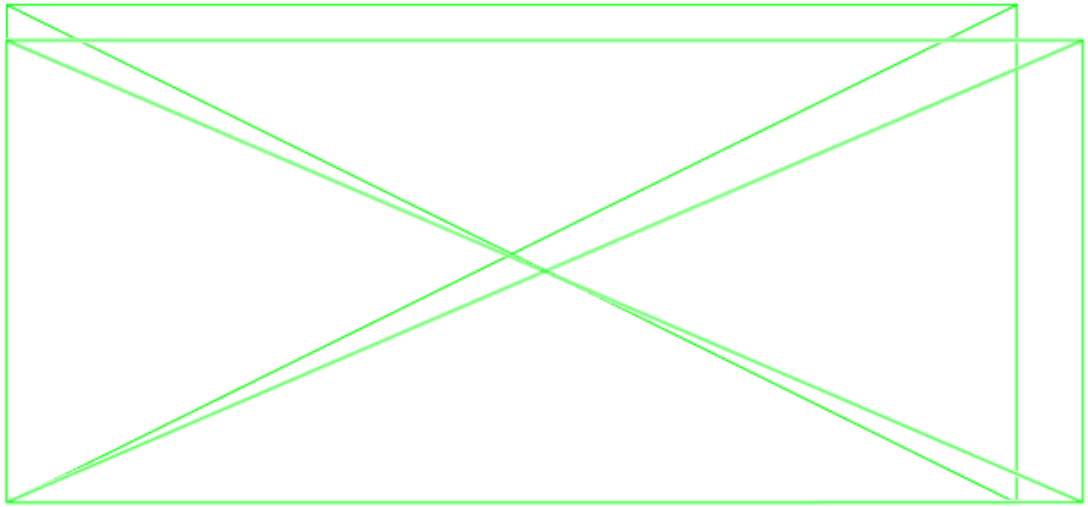
# Modelling Continuous Structures





SYSTEM STIFFNESS MATRIX

```
 BLOCK NUMBER          1
 FIRST EQUATION        1
 LAST EQUATION         7

   1          1
     0.25096E+07
   2          1          2
     0.55385E+07  0.75000E+06
   3          1          3
     0.10038E+08 -0.25096E+07
   4          1          2          3          4
     0.13846E+07 -0.75000E+06 -0.13846E+07 -0.28846E+05
   5          1          2          3          4          5
     0.25096E+07  0.75000E+06 -0.25096E+07 -0.75000E+06  0.10529E+07
   6          1          2          3          4          5
     0.13846E+07 -0.28846E+05 -0.11538E+06  0.75000E+06 -0.13846E+07
   7          1          2          3          4          5          6
     0.25096E+07 -0.75000E+06 -0.10529E+07  0.28846E+05 -0.25096E+07  0.75000E+06

MAXIMUM DIAGONAL STIFFNESS MATRIX VALUE =  0.100385E+08 (       3)
MINIMUM DIAGONAL STIFFNESS MATRIX VALUE =  0.138462E+07 (       4)
```

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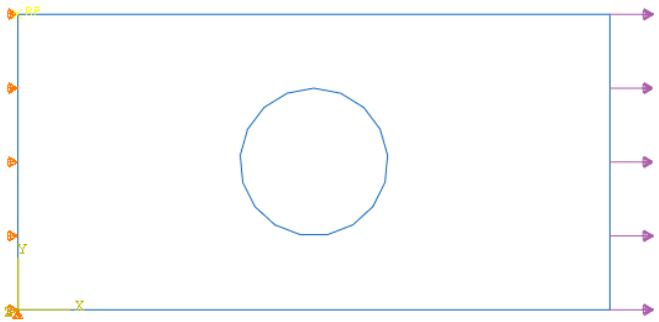
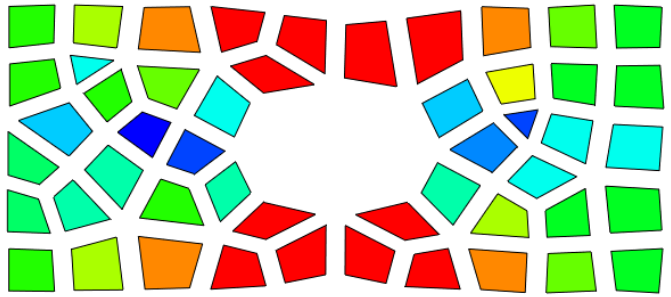
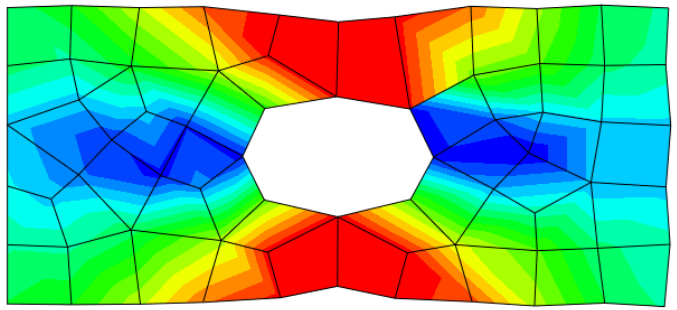
# Modelling Continuous Structures - stresses

- Stresses are calculated from the deformed element shapes.
  - They are discontinuous over the model.
  - The smooth stress contours are achieved by averaging the stress results at the nodes



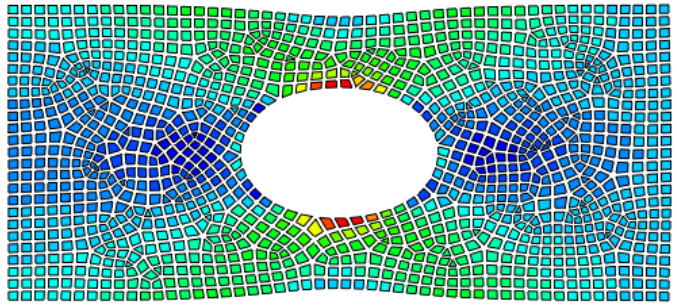
Unaveraged stresses

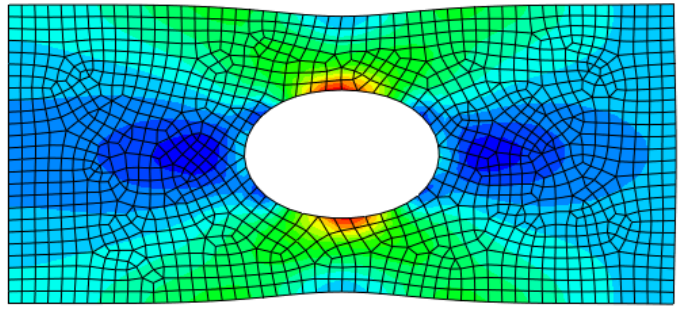




Averaged stresses

Unaveraged stresses





Averaged stresses

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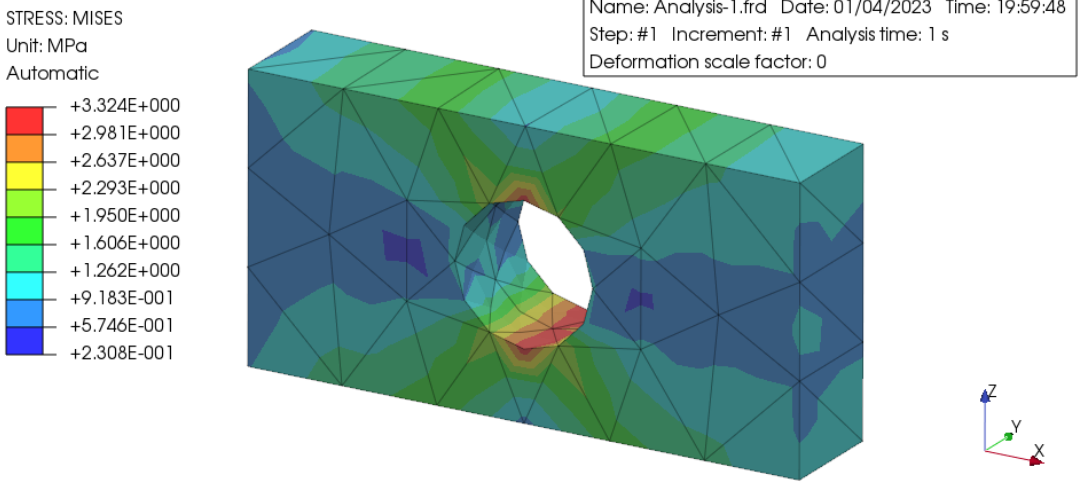
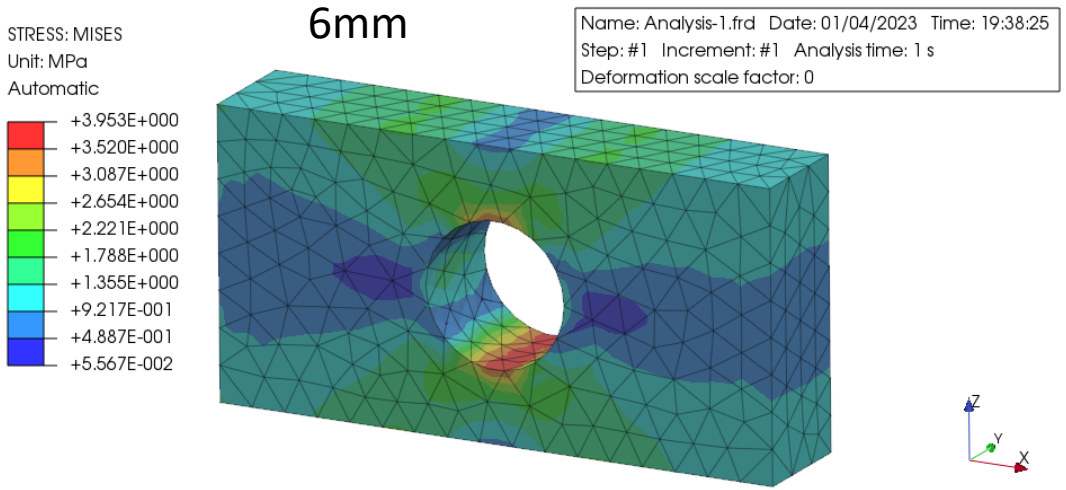
# Mesh convergence

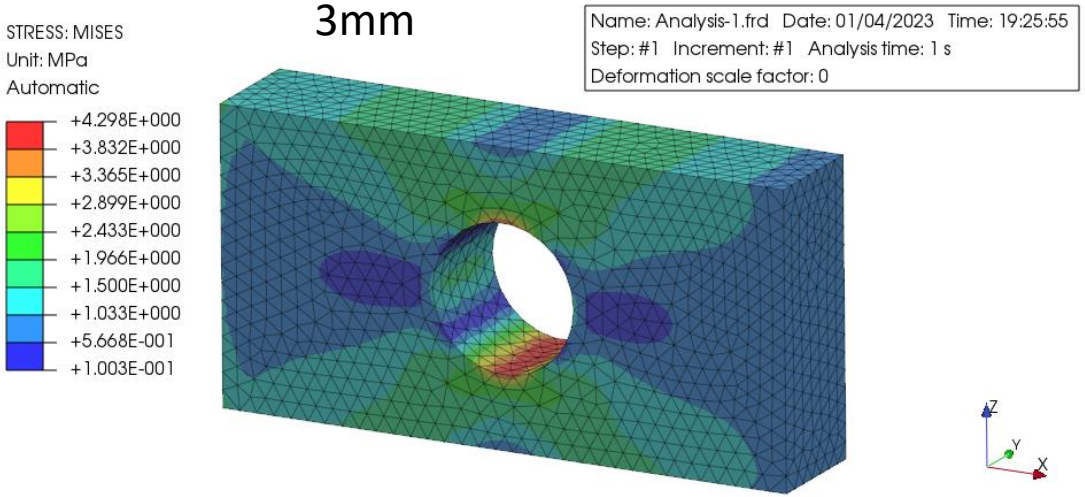
18mm



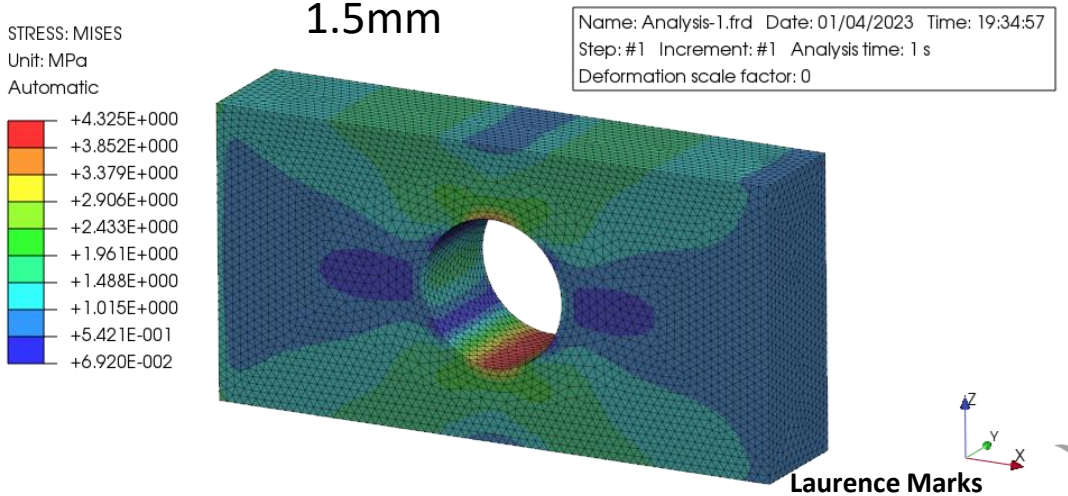

6mm




3mm




1.5mm

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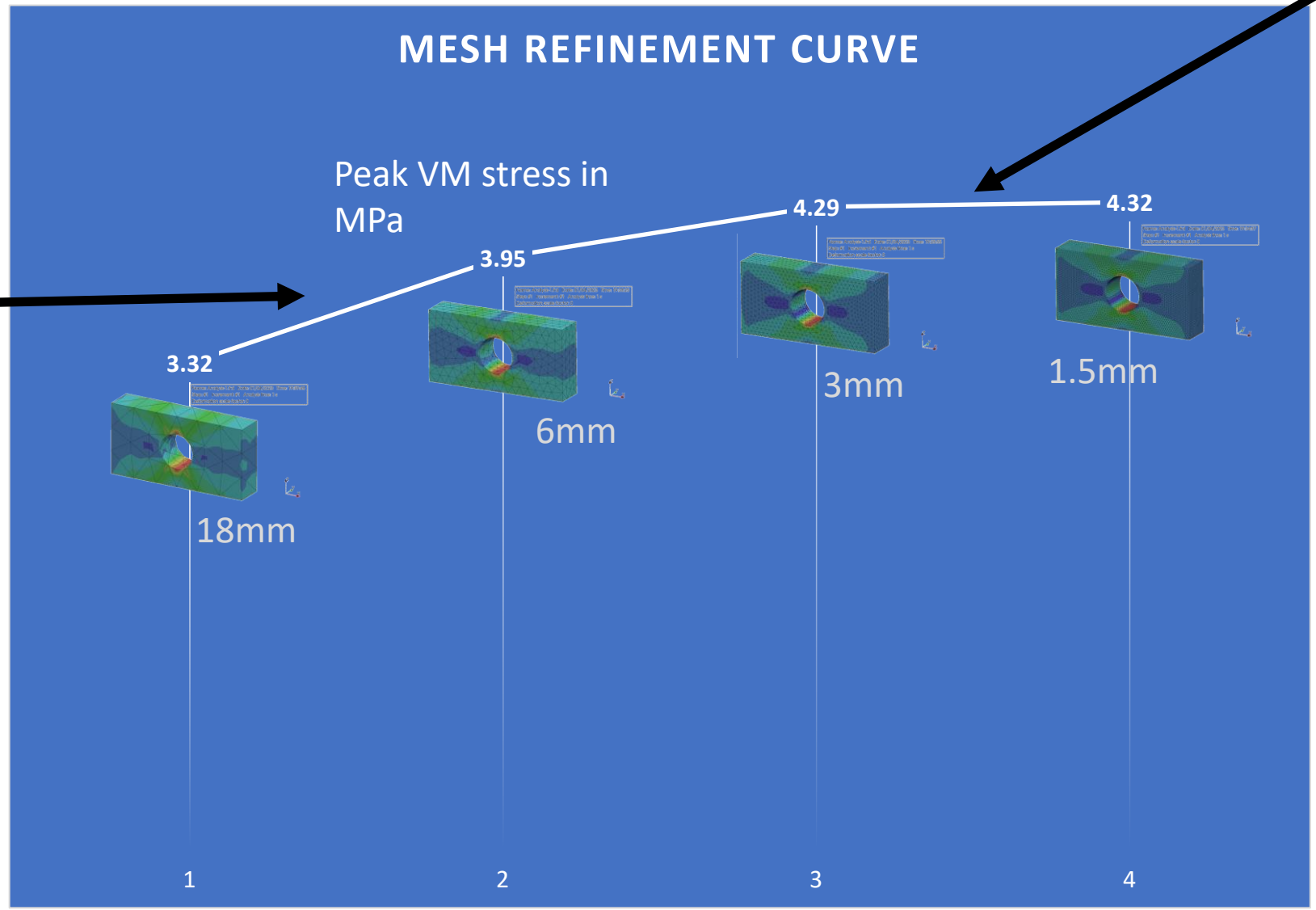
# Mesh convergence

Here changes of element size change the answer




Here changes of element size don't change the answer

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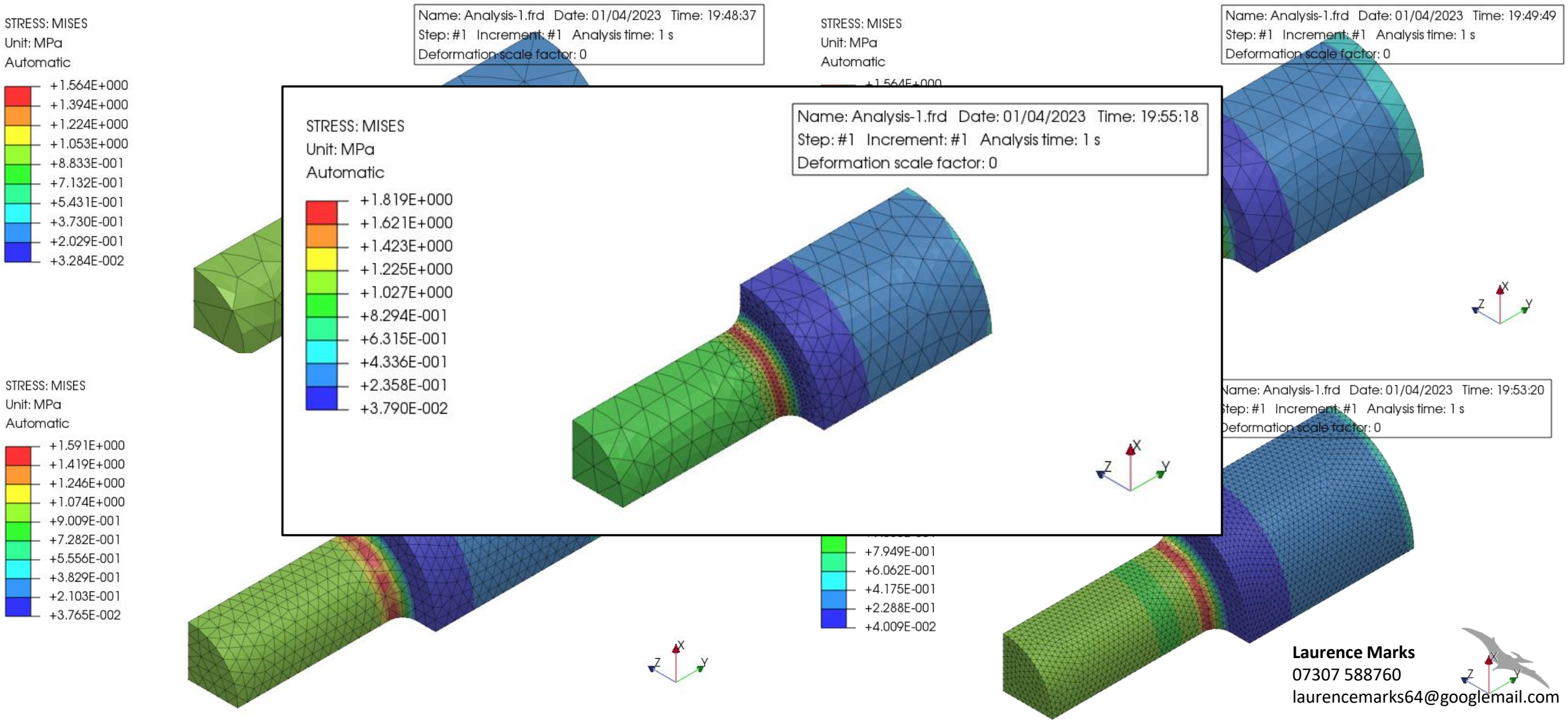
# Local Mesh refinement

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